Radiowave Propagation

Antennas over Flat Earth

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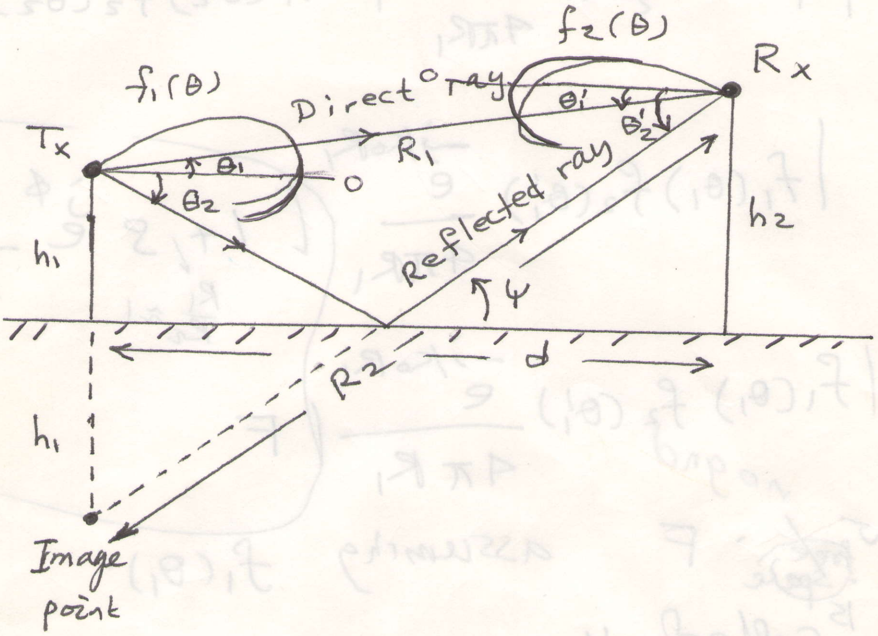
1. ANTENNAS LOCATED OVER A FLAT EARTH

Antennas Located over a flat Earth:



$f_i(\theta)$: the radiated field strength patterns of antenna i.

The direct ray that reaches the Receiving antenna produces a voltage propotional to

$$v_d = f_1(\theta_1)\, f_2(\theta_1')\, \frac{e^{-jk_0R_1}}{4\pi R_1}$$

The voltage received due to the reflected ray is proportional to:

$$v_r = f_1(\theta_2)\, f_2(\theta_2')\left(\rho e^{j\phi}\right) \frac{e^{-jk_0R_2}}{4\pi R_2}$$

$\rho e^{j\phi}$: reflection coefficient at the ground.

The total received voltage is proportional to

$$v = v_d + v_r$$

$$= \left| f_1(\theta_1) f_2(\theta_1') \frac{e^{-jk_0 R_1}}{4\pi R_1} + f_1(\theta_2) f_2(\theta_2') \rho e^{j\phi} \frac{e^{-jk_0 R_2}}{4\pi R_2} \right|$$

$$= \left| f_1(\theta_1) f_2(\theta_1') \frac{e^{-jk_0 R_1}}{4\pi R_1} \left[1 + \underset{\frac{R_1}{R_2} \approx 1}{\rho}\, e^{j\phi} \frac{f_1(\theta_2) f_2(\theta_2') e^{-jk_0(R_2 - R_1)}}{f_1(\theta_1) f_2(\theta_1')} \right] \right|$$

$$= \left| f_1(\theta_1) f_2(\theta_1') \frac{e^{-jk_0 R_1}}{4\pi R_1} \right| F$$

$v \cong v_{\substack{\text{no gnd} \\ \text{free space}}} \cdot F$ assuming $f_1(\theta_1)$

F : is called the path gain factor

If the antenna radiation patterns are approx. constant over the range of angles involved $\Rightarrow f_1(\theta_1) = f_1(\theta_2)$, $f_2(\theta_1') = f_2(\theta_2')$

$$F = \left| 1 + \rho e^{j\phi} e^{-jk_0(R_2 - R_1)} \right|$$

$$R_1 = \left[d^2 + (h_2 - h_1)^2 \right]^{\frac{1}{2}}$$

$$= d \left[1 + \left(\frac{h_2 - h_1}{d} \right)^2 \right]^{\frac{1}{2}}$$

$$\approx d \left(1 + \frac{1}{2} \left(\frac{h_2 - h_1}{d} \right)^2 \right), \quad \frac{h_2 - h_1}{d} \ll 1$$

$$= d + \frac{(h_2 - h_1)^2}{2d}$$

Similarly,

$$R_2 \approx d + \frac{(h_2 + h_1)^2}{2d}$$

$$R_2 - R_1 \approx \frac{2h_1 h_2}{d}$$

For perfect conducting plane (earth) $\rho e^{j\phi} = -1$
(horizontal polarization)

$$F = \left| 1 - e^{-jk_0 \frac{2h_1h_2}{d}} \right| =$$

$$= \left| 1 - \cos\left(\frac{2k_0 h_1 h_2}{d}\right) + j \sin\left(\frac{2k_0 h_1 h_2}{d}\right) \right|$$

$$= \left[\left(1 - \cos \frac{2k_0 h_1 h_2}{d}\right)^2 + \sin^2\left(\frac{2k_0 h_1 h_2}{d}\right) \right]^{\frac{1}{2}}$$

$$= \left[1 + \underbrace{\cos^2\left(\frac{2k_0 h_1 h_2}{d}\right) + \sin^2\left(\frac{2k_0 h_1 h_2}{d}\right)}_{1} - 2\cos\left(\frac{2k_0 h_1 h_2}{d}\right) \right]^{\frac{1}{2}}$$

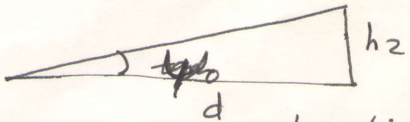
$$= \left[2 - 2\cos\left(\frac{2k_0 h_1 h_2}{d}\right) \right]^{\frac{1}{2}} = \sqrt{4 \sin^2\left(\frac{k_0 h_1 h_2}{d}\right)}$$

$$= 2 \left| \sin\left(\frac{k_0 h_1 h_2}{d}\right) \right|$$

The maximum value of F is 2 (i.e., double the free space field strength for $\alpha = \infty$).

This relationship is usually plotted in the form of a coverage diagram showing the variation of F with ~~the and~~ h_2 and d for given values of h_1 and λ_0 ($k_0 = \frac{2\pi}{\lambda_0}$).

F is max when



$\tan \psi_0 = \frac{h_2}{d}$

$$\frac{2\pi}{\lambda_0} \frac{h_1 h_2}{d} = \pm \frac{\pi}{2} + n\pi$$

$$\frac{h_2}{d} = \frac{\lambda_0}{h_1}\left(\frac{1}{4} + \frac{n}{2}\right); \quad n = 0, \pm 1, \pm 2, \ldots$$

and F is minimum when

$$\frac{2\pi}{\lambda_0} \frac{h_1 h_2}{d} = n\pi \quad ; \quad n = 0, 1, 2, \ldots$$

$$\frac{h_2}{d} = \frac{\lambda_0}{h_1} \frac{n}{2}$$

proportional to field strength

The coverage diagram (this constant represents) is a plot of F/r = constant ($r \approx d$) which represents the same signal level that would be obtained at a distance of multiple or fractional multiple of a convenient free space range r_f ($F/r = \frac{m}{r_f}$, $m = 1, \sqrt{2}, 2, \ldots, \frac{1}{\sqrt{2}}, \frac{1}{2}, \ldots$ i.e, 3 dB difference between curves).

Note that the free space signal varies as $\frac{1}{r_f}$ whereas the ~~the~~ total (direct + reflected) signal varies as $\frac{F}{r}$.

In polar coordinates, d can be viewed as the radial distance and the elevation angle ψ_0 ($\tan\psi_0 = \frac{h_2}{d}$) as the angular position.

How to plot the Coverage Diagram:

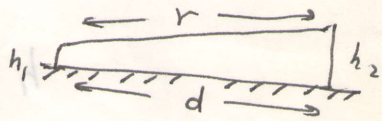
Example

let the reference free space range $= r_f = 2\,Km$, and $h_1 = 100\,\lambda_0$, then:

$$F = 2\left|\sin\left(2\pi \frac{h_1}{\lambda_0}\frac{h_2}{d}\right)\right| = m\,\frac{d}{d_f}\ ,\ r_f \approx d_f$$

$$\frac{2\pi h_1}{\lambda_0} = 200\pi$$

let $m = 1$



$$\left|\sin\left(200\pi\frac{h_2}{d}\right)\right| = \frac{1}{2}\,\frac{d}{d_f}$$

$h_1, h_2 \ll d$

$\Rightarrow r \approx d$

$$= \frac{1}{2}\,\frac{d}{2000\,m} = \frac{d}{4000}$$

the max. value of $d = 4000$ ($\sin = 1$)

$$\therefore\ 200\cancel{\pi}\,\frac{h_2}{4000} = \frac{\cancel{\pi}}{2} + n\cancel{\pi}\ ,\ n = 0, 1, 2, \ldots$$

$$h_2 = 20\left(\tfrac{1}{2} + n\right) = 10 + \underbrace{20n}$$

$\Rightarrow$ angular separation

$\therefore$ the max. value of the received signal occurs at

$$d = 4\,Km\ \ \&\ \ h_2 = 10, 30, 50, 70, \ldots\ m$$

The angular separation between successive loops is

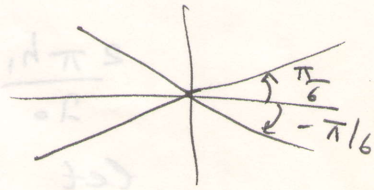
$$\tan\varphi_0 = \frac{h_2}{d} = \frac{20}{4000} = \frac{1}{200}\ \text{rad} = 0.3^\circ$$

Consider $d = 2\,Km \Rightarrow$ what values of h_2 that will give same signal level as the free space reference range (2km) would give?

$$\left| \sin\left(200\pi \frac{h_2}{d}\right)\right| = \frac{1}{2}\,\frac{d}{d_f}$$

$$\left| \sin\left(200\pi \frac{h_2}{2000}\right)\right| = \frac{1}{2}\,\frac{2000}{2000} = \frac{1}{2}$$

$$\therefore \quad 200\cancel{\pi}\,\frac{h_2}{2000} = \pm\frac{\cancel{\pi}}{6} + n\cancel{\pi},$$



$$h_2 = 10\left(\pm\frac{1}{6} + n\right) = \pm\frac{10}{6} + 10n$$

$$h_2 = 1\tfrac{2}{3},\ 8.3,\ 11.7,\ 18.3,\ 21.7,\ 28.3,\ 31.7, \ldots m$$

$\uparrow \frac{10}{6}$ $\quad \uparrow -\frac{10}{6}+10$ $\quad \uparrow \frac{10}{6}+10$ $\quad \uparrow -\frac{10}{6}+20$ $\quad \uparrow \frac{10}{6}+20$ $\quad \uparrow -\frac{10}{6}+30$ $\quad \uparrow \frac{10}{6}+30$

Consider $d = 3\,Km \Rightarrow$ find h_2 that will give same signal level as that of free space range of 2km

$$\left| \sin 200\pi \frac{h_2}{3000}\right| = \frac{1}{2}\,\frac{3000}{2000} = 0.75$$

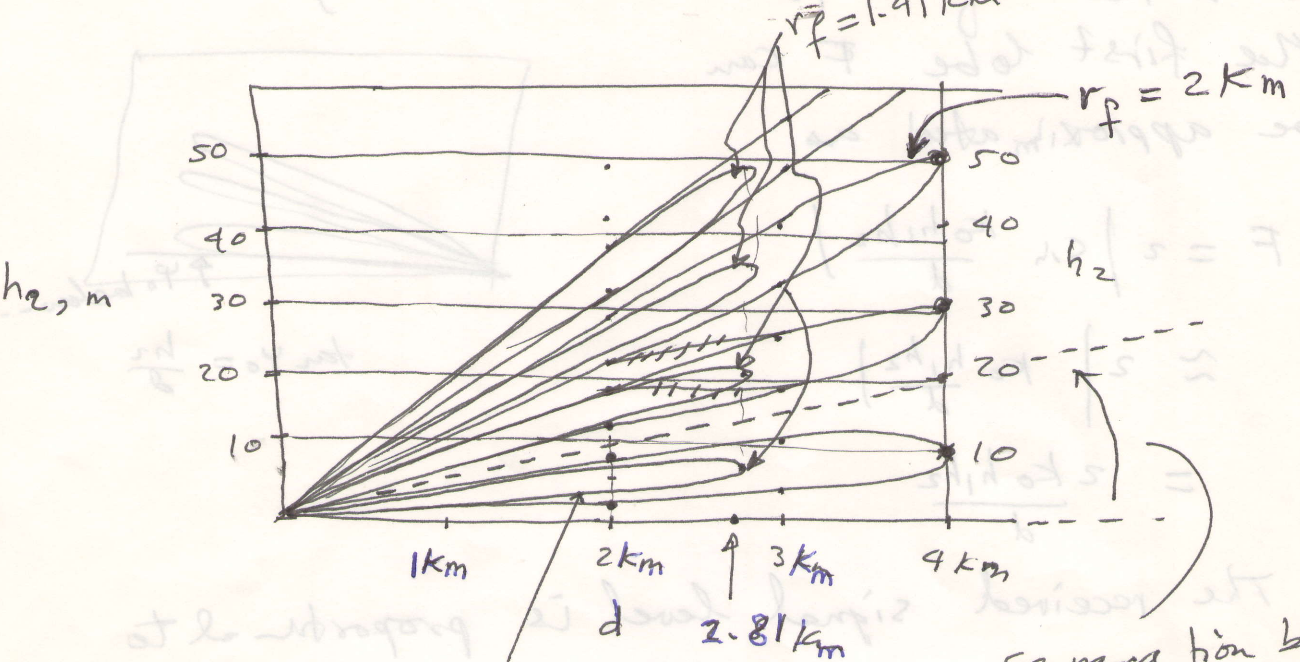
$$200\pi \frac{h_2}{3000} = \pm 0.85\,\text{rad} + n\pi, \quad n = 0, 1, 2, 3, \ldots$$

$$h_2 = \frac{15}{\pi}(\pm 0.85 + n\pi) = \pm 4 + 15n$$

$$h_2 = 4,\ 11,\ 19,\ 26,\ 34,\ 41,\ 49,\ 56,\ 64,\ 71,\ 79$$

$\uparrow -4+15$ $\quad \uparrow 4+15$ $\quad \uparrow -4+30$ $\quad \uparrow 4+30$ $\quad \uparrow -4+45$ $\quad \uparrow 4+45$ $\quad \uparrow -4+60$ $\quad \uparrow 4+60$ $\quad \uparrow -4+75$

plotting these values produces the following lobes



$h_1 = 100\lambda_0$ $\quad r_f = 1.41$ km

$r_f = 2$ km

$$\tan \psi_0 = \frac{20}{4000} = \frac{1}{200}$$

$$\therefore \psi_0 \simeq \frac{1}{200}\text{ rad} = 0.3^\circ$$

$(\tan \psi_0 \simeq \psi_0, \ \psi_0 \ll 1)$

In the equation:

$$F = 2\left| \sin\left(2\pi \frac{h_1}{\lambda_0} \frac{h_2}{d} \right) \right| = m \frac{d}{d_f}, \quad r_f = 2\text{ km}$$

consider $m = \sqrt{2}$ (i.e 3dB higher signal contour)

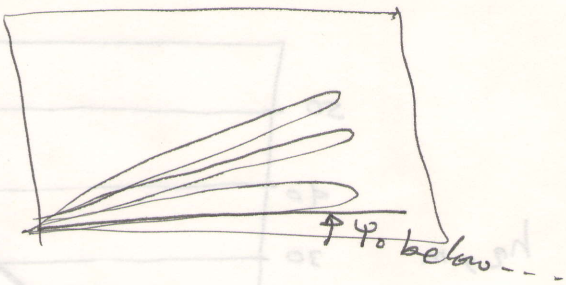
$$\therefore F = \sqrt{2}\,\frac{d}{d_f} = \frac{d}{\left(\frac{d_f}{\sqrt{2}}\right)} = \frac{d}{d_f'}$$

$$\therefore d_f' = \frac{d_f}{\sqrt{2}} = \frac{2\text{ km}}{\sqrt{2}} = \sqrt{2}\text{ km} = 1.41\text{ km}$$

$$\therefore \text{max. } d = 2 d_f' = 2.81\text{ km}$$

the loop corresponding to this reference is shown as the small loop in the figure above.

when the angle φ_0 is considerably below the the first lobe F can be approximated as



$$\tan\varphi_0 = \frac{h_2}{d}$$

$$F = 2\left|\sin\frac{k_0 h_1 h_2}{d}\right|$$

$$\simeq 2\left|k_0\frac{h_1 h_2}{d}\right|$$

$$= \frac{2k_0 h_1 h_2}{d}$$

The received signal level is proportional to

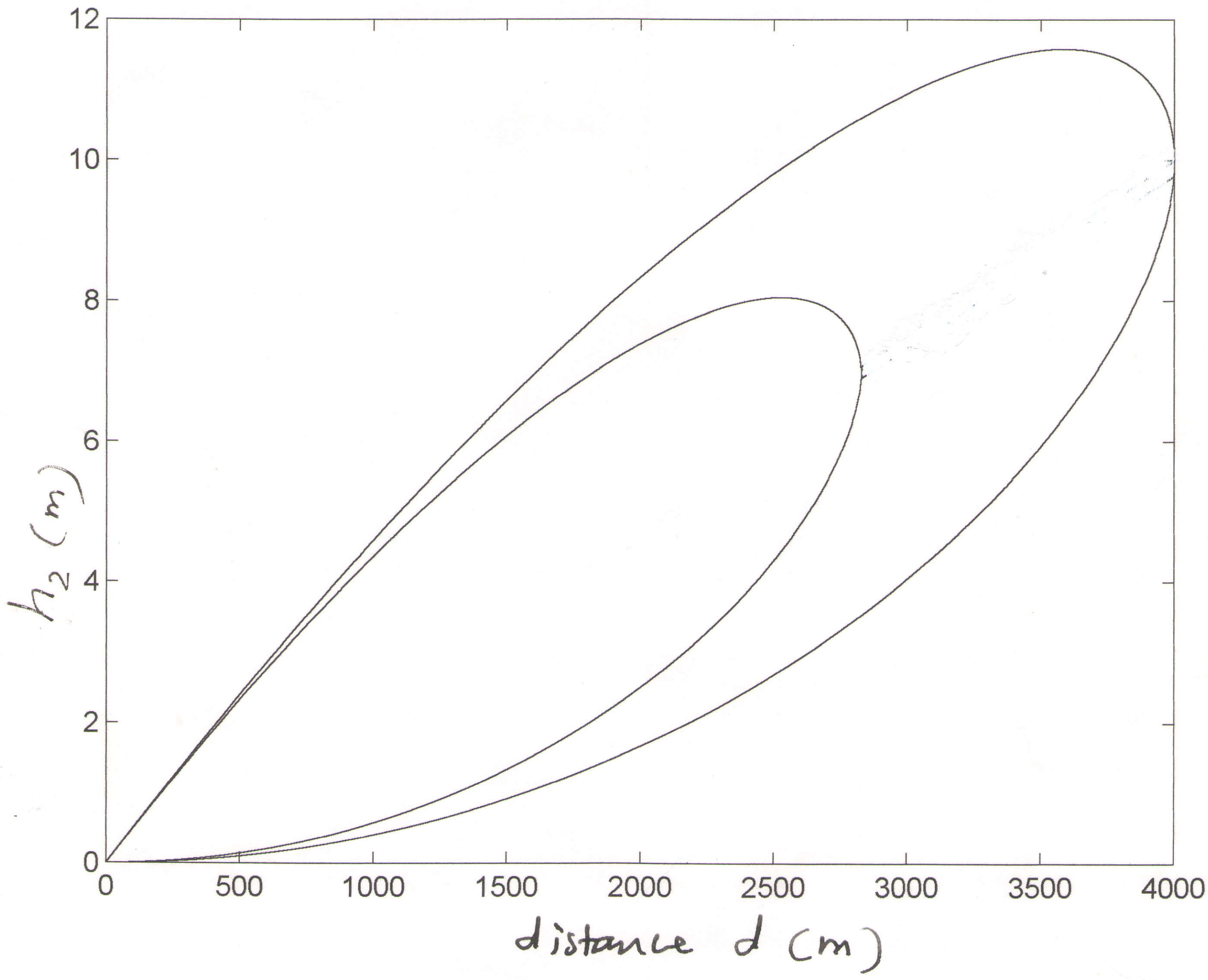
$$\frac{F}{r} \simeq \frac{F}{d} = \frac{2k_0 h_1 h_2}{d^2}$$

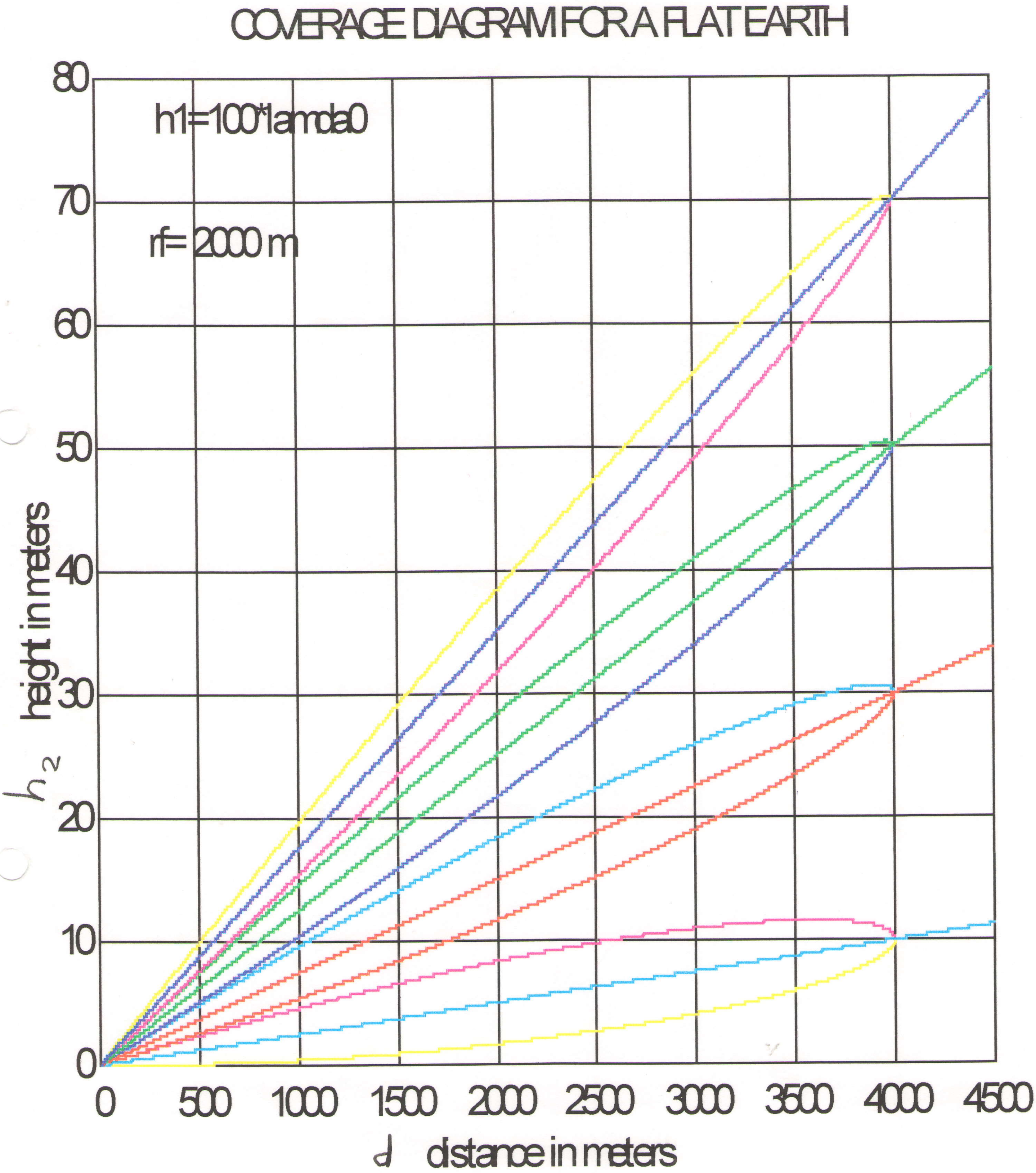
$\therefore$ the signal level in this region varies as the inverse square of the distance, thus reducing the maximum useful range quite severely.

The equation above is valid for $\rho = -1$. This is ~~true~~ for $\varphi_0 < 1^\circ$ (~~grazing~~ vertical incidence) and is good approximation ~~at~~ for horizontal polarization for $\varphi_0 < 10^\circ$. However, if $\varphi_0 > 10^\circ$ for horizontal pol. or $\varphi_0 > 1^\circ$ for vertical polarization, ρ cannot be approximated by -1 & the curves above are not valid.

Matlab code that plots coverage diagram of a flat earth 11

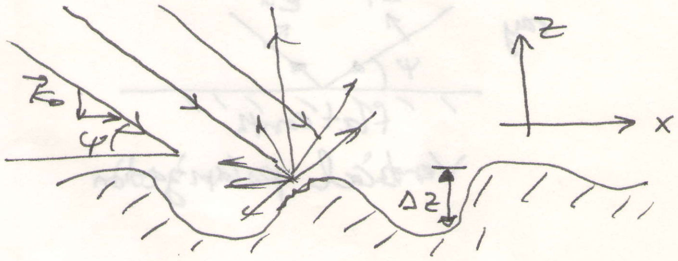
```
 for d=1:4000
h(d)=d*(1/(200*pi))*asin(sqrt(2)*d/(4000));
z(d)=2*d*6.99/(2828)-h(d);
h1(d)=d*(1/(200*pi))*asin(d/(4000));
z2(d)=2*d*10/(4000)-h1(d);

h2(d)=d*40/4000-h(d);
z1(d)=2*d*30/(4000)-h2(d);
h3(d)=d*60/4000-h(d);
z3(d)=2*d*50/4000-h3(d);
h4(d)=d*80/4000-h(d);
z4(d)=2*d*70/4000-h4(d);
end
for d=1:4500

y1(d)=10*d/4000;
y2(d)=30*d/4000;
y3(d)=50*d/4000;
y4(d)=70*d/4000;
end
x=1:4500;
 d=1:4000;

 plot(d,h(d),d,z(d),d,h1(d),d,z2(d),d,h2(d),d,z1(d),d,h3(d),d,z3(d
),d,h4(d),d,h5(d),'-')
```


12
10
8
6
4
2
0
0
500
1000
1500
2000
2500
3000
3500
4000
h_2 (m)
distance d (m)


COVERAGE DIAGRAM FOR A FLAT EARTH
h1=100*lamda0
rf= 2000 m
h_2 height in meters
d distance in meters
0
10
20
30
40
50
60
70
80
0
500
1000
1500
2000
2500
3000
3500
4000
4500

Nonflat earth

If the region over which the wave is reflected is not flat (rough surface), the field will be scattered and the specular component, and hence ρ, is reduced in value.



rough surface

The incident wave has a propagation factor

$$e^{-j\vec{k}_0 \cdot \vec{r}},$$

$$\vec{k}_0 = k_0 \cos\psi \, \hat{x} - k_0 \sin\psi \, \hat{z}$$

$$\vec{r} = x\hat{x} + z\hat{z}$$

$$\vec{k}_0 \cdot \vec{r} = x k_0 \cos\psi - z k_0 \sin\psi$$

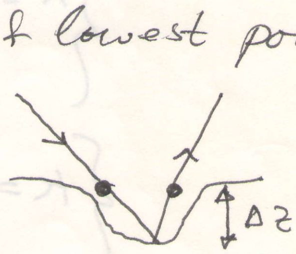
$$e^{-j\vec{k}_0 \cdot \vec{r}} = e^{jzk_0 \sin\psi - jxk_0 \cos\psi}$$

The component of k_0 in the vertical direction is $k_0 \sin\psi = \dfrac{2\pi}{\lambda_v}$; λ_v: wave length in vertical direction (z dir.)

$$\lambda_v = \frac{\lambda_0}{\sin\psi}, \quad \Delta z \leq \frac{\lambda_v}{10} \text{ (smooth surface)}$$

If the height of the irregularity is Δz (Fig. above), the phase difference between the wave reflected from the highest & lowest points of the irregularity is

$$\Delta\phi = 2\,(k_0 \sin\psi)\,\Delta z$$



Consider the boundary between rough and smooth surface to be $\frac{\lambda_v}{10}$, then

$$\Delta\phi_m = 2(k_0 \sin\psi)\frac{\lambda_v}{10},$$

but $\lambda_v = \frac{\lambda_0}{\sin\psi}$

$$\Delta\phi_m = 2\left(\frac{2\pi}{\cancel{\lambda_0}}\cancel{\sin\psi}\right)\frac{\cancel{\lambda_v}}{10} = 0.4\pi$$

<u>example</u> $\lambda_0 = 3m\ (f = \cancel{1}00 MHz),\ \psi = 2^\circ$

$\lambda_v = 86_m \Rightarrow \frac{\lambda_v}{10} = \frac{\lambda_0}{10\sin\psi} = \frac{3}{10(\psi)}$, $\sin\psi \simeq \psi,\ \psi \ll 1$

$= \frac{3}{2}\frac{60}{10} = 9m$. 8.6m? $\psi = 2^\circ = 2\frac{3.14}{180} \simeq \frac{2}{60}$ rad

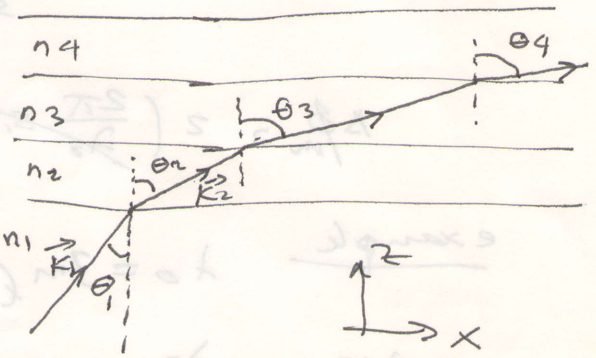
for $\psi = 1^\circ \rightarrow \lambda_v = 171.9m \rightarrow 0.1\lambda_v = 17.2m$

Thus, a height of 9m of irregularities can be considered smooth surface for such frequency and grazing angle ψ. As a conclusion, for the lower frequencies the surface appears to be smooth while at the μwave frequencies, most surfaces would be rough & the reflection coeff. would be smaller than those given by Fresnel formulas

$$\frac{3}{10\times\frac{1}{30}} = \frac{39\times3}{19} = 9$$

The effect of change in the index of refraction n was not taken into account in the interference formulas for the flat earth.

$n = \sqrt{\varepsilon_r}$



$\therefore$ n decreases monotonically with height in general (exceptions occur where n increases with height in some regions which cause the ducting phenomenon).

$n_1 > n_2 > n_3 > n_4 > \cdots$

The boundary condition at each interfaces requires that

$$\vec{k}_i \cdot \hat{x} = \vec{k}_{i+1} \cdot \hat{x}, \quad i = 1, 2, 3, \ldots$$

$$k_i \sin\theta_i = k_{i+1} \sin\theta_{i+1}$$

$$\cancel{k_0} \sqrt{\varepsilon_{r_i}} \sin\theta_i = \cancel{k_0} \sqrt{\varepsilon_{r_{i+1}}} \sin\theta_{i+1}$$

$$n = \sqrt{\varepsilon_r}$$

$$n_i \sin\theta_i = n_{i+1} \sin\theta_{i+1}$$

since $n_{i+1} < n_i$

$$\theta_{i+1} > \theta_i$$

and the ray band in a downward direction. ~~This~~ For propagation over spherical earth, this ray curvature extends the radio horizon beyond the geometrical horizon.

The effect of ray curvature over a spherical earth can be taken into account by increasing the radius of the earth (8497 km a_e) and considering the rays to propagate along straight lines for standard index of refraction profile.

~~Maximum~~

The interference formulas for flat earth are not generally valid for distances approaching the maximum horizontal line-of-sight range. d_M.

$$R^2 = (a_e + h_1)^2 - a_e^2$$

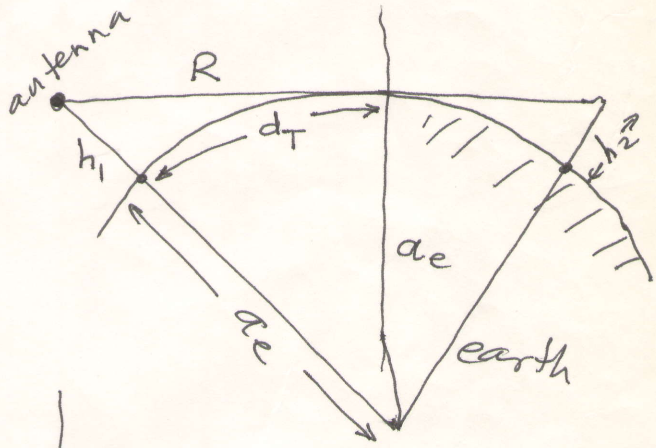
$$= 2a_e h_1 + h_1^2$$

since $h_1 \ll a_e$

$$R^2 \approx 2a_e h_1 , \quad R \approx d_T$$

$$R \cong \sqrt{2 a_e h_1}$$

$$d_T \approx \sqrt{2 a_e h_1}$$



a_e has to be used here since the rays are considered straight.

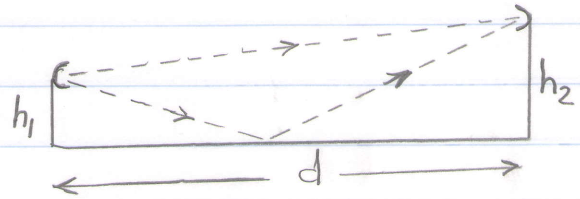
$\therefore$ The maximum line-of-sight distance between two antenna over spherical earth may be expressed as

$$d_M = \sqrt{2a_e h_1} + \sqrt{2 a_e h_2} \quad , \text{ meters}$$

For Line-of-Sight communications (space wave)

$$d_M = \sqrt{2a_e h_1} + \sqrt{2a_e h_2}$$

1. Free-space Zone:



Ground reflected signal is neglected. Thus, Friis transmission formula may be used.
Maximum range for this zone is obtained from:

$$\frac{2 h_2 d_{max}}{d_{max}^2 - (h_2^2 - h_1^2)} = \tan 5^\circ$$

Or

$$d_{max} = \frac{h_2}{\tan 5^\circ} + \sqrt{\left(\frac{h_2}{\tan 5^\circ}\right)^2 + (h_2^2 - h_1^2)}$$

$$\therefore \quad 0 < d < d_{max}$$

2. Flat-Earth Intesfeence Zone $\left(F = 2\left|\sin\left(\frac{k_0 h_1 h_2}{d}\right)\right|\right)$

$$d_{max} < d < 0.3\, d_M$$

3. Curved-Earth Interference Zone (Design Curves)

$$0.3\, d_M < d < 0.75 d_M$$

4. Tangent-Ray Zone

$$0.75\, d_M < d < 1.25\, d_M$$

5. Diffraction Zone

$$d > 1.25\, d_M$$

Flat Earth

Example (Flat Earth)

$f = 937.5$ MHz, $h_1 = 16$ m, $d = 2$ km

Find receiver heights for maximum received signal.

Solution:

received signal v

$$v \propto \frac{F}{r} \approx \frac{F}{d} \quad \text{note that } h_1, h_2 \ll r$$

since d is fixed, to maximize v, maximize F

$$F_{max} = 2\left|\sin\left(2\pi \frac{h_1}{\lambda_0}\frac{h_2}{d}\right)\right| = 2$$

$$2\pi \frac{h_1 h_2}{\lambda_0 d} = \frac{\pi}{2} + n\pi \;;\; n = 0, 1, 2, 3, \ldots$$

$$\lambda_0 = \frac{3\times10^{10}}{937.5\times10^6} = 32 \text{ cm}$$

$$h_2 = \frac{\lambda_0 d}{2\pi h_1}\left(\frac{\pi}{2} + n\pi\right)$$

$$= \frac{0.32 \times 2000}{2\cancel{\pi}\times 16}\left(\frac{\cancel{\pi}}{2} + n\cancel{\pi}\right)$$

$$= \overset{20}{\cancel{3.36}}\left(\frac{1}{2} + n\right) = 10 + 20n \;;\; n = 0, 1, 2, \ldots$$

$$= 10, 30, 50, 70, 90, \ldots \text{ m}.$$

(max. distance) on the earth

(Flat Earth)

at which received signal is a) max. b) zero. finite

Example

$f = 1\,GHz$, $h_1 = 21\,m$, $h_2 = 18\,m$

Find the farthest three locations of the receiver (distance between receiver & transmitter) to receive

(a) max. radiation (incident and reflected waves are in phase)

(b) no radiation — 1st min. three heights of receiver

(c) For $d = 2\,km$, find h_2 to obtain same radiation as that of ~~free space~~ when receiver is in free space (no ground) at 3 km distance.

Solution:

(a) $$\lambda_0 = \frac{3\times10^{10}}{10^9} = 30\ cm$$

$$F_{max} = 2 = 2\left|\sin\left(\frac{2\pi h_1 h_2}{\lambda_0 d}\right)\right|$$

$$\frac{2\pi h_1 h_2}{\lambda_0 d} = \frac{\pi}{2} + n\pi \qquad n = 0, 1, 2, \ldots$$

$$d = \left(\frac{1}{\frac{1}{2}+n}\right)\frac{2h_1 h_2}{\lambda_0}$$

$$= \frac{2}{1+2n}\;\frac{2\times21\times18}{0.3} = \frac{5040}{1+2n}$$

$$d_{max} = 5040\ m;\ n = 0$$

(b) $$\sin\left(\frac{2\pi h_1 h_2}{\lambda_0 d}\right) = 0$$

$$\frac{2\pi h_1 h_2}{\lambda_0 d} = n\pi$$

$$d = \frac{2h_1h_2}{\lambda_0 n} = \frac{2\times 21\times 18}{0.3\times 1} \quad ; \quad n=1 \text{ (farthest distance)}$$

$$= 2520 \text{ m}$$

(c) $$\frac{F}{d} = \frac{E}{2000\text{ m}} = \frac{1}{3000}$$

$$F = \frac{2000}{3000} = \left| 2 \sin\left(\frac{2\pi}{\lambda_0}\frac{h_1h_2}{d}\right)\right| = \frac{2}{3}$$

$$\sin \frac{2\pi}{\lambda_0}\frac{h_1h_2}{d} = \frac{1}{3}$$

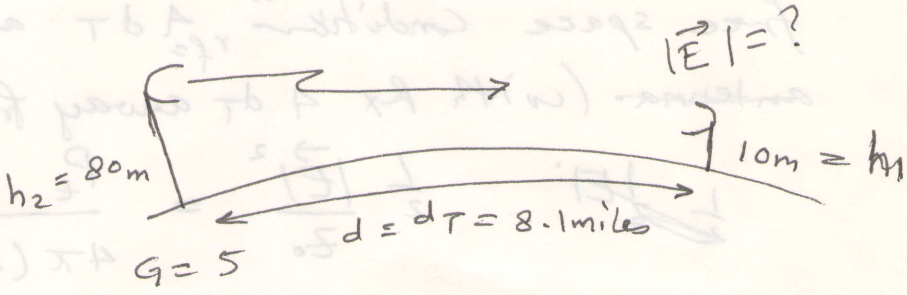
$$\frac{2\pi}{\lambda_0}\frac{h_1h_2}{d} = 0.3398\ldots + n\pi \quad ; \quad n = 0, 1, 2, \ldots$$

$$h_2 = \frac{\lambda_0 d}{2\pi h_1}(0.3398 + n\pi)$$

$$h_{2\,min} = \frac{\lambda_0 d}{2\pi h_1} \times 0.3398 \quad ; \quad n=0$$

$$= 1.545\text{ m}, \ 15.83\text{ m}, \ 30.12\text{ m}$$

Example 6.3 FM Communication Link:



$G = 5$

$P_{tx} = 500W$

$f = 100 MHz \ (\lambda = 3m)$

$$\nu = \frac{h_1^{3/2}}{1030\lambda_0} = \frac{10\sqrt{10}}{(1030)(3)} = 0.01$$

These is no coverage diagram with $\nu = 0.01$

$$d_M = d_{T_1} + d_{T_2}$$

$$= \sqrt{2a_e h_1} + \sqrt{2a_e h_2}$$

$$= \sqrt{2h_1} + \sqrt{2h_2} \ \text{mi}, \quad h_1 \ \& \ h_2 \text{ in feet}$$

$$\cong \sqrt{30 \times 2} \times \sqrt{240 \times 2} \qquad a_e = 8497 \text{ km}$$

$$= \sqrt{60} + \sqrt{480} = 8 + 22 \cong 30 \text{ miles}$$

$$13.036 + 36.87 = 50 \text{ km}$$

20x20
21
21
21
42
441

$$d_T = \sqrt{2a_e h_1} = 8.1 \text{ miles} = 13.036 \text{ km}$$

$$d = d_T = 8.1 \text{ miles} < \frac{d_M}{3}$$

$\therefore$ use flat earth formula $= 2\left|\sin \frac{2\pi}{\lambda_0}\frac{h_1 h_2}{d}\right|$

$$F = 2\left|\sin\left(\frac{\pi}{2}\nu\frac{h_2/h_1}{d/d_T}\right)\right| = 2\left|\sin\left(\frac{\pi}{2}0.01\frac{8}{1}\right)\right|$$

$$\cong \cancel{2} \times \frac{\pi}{\cancel{2}} \times 0.01 \times 8 = 0.25 = \frac{d}{r_f} = \frac{d_T}{r_f} \ ; \ d = d_T$$

$$\therefore r_f = 4 d_T$$

∴ the field strength at the receiving antenna is the same as ~~the~~ ~~would be~~ that = obtained under free space conditions with $r_f = 4d_T$ ~~away from the Tx antenna~~ (with Rx $4 d_T$ away from Tx).

$$\cancel{\tfrac{1}{2}|E|} \quad \frac{1}{2}\frac{|\vec{E}|^2}{Z_0} = \frac{P_t}{4\pi(4d_T)^2}\, G$$

$$|\vec{E}| = \sqrt{\frac{2 Z_0 P_t G}{4\pi}}\; \frac{1}{4 d_T}$$

$$= \sqrt{\frac{2\times 1\cancel{20}^{30}\pi \times 500\times 5}{\cancel{4\pi}}}\; \frac{1}{4\times 13.03\times 10^3_m}$$

$$= \frac{\sqrt{60\times 500\times 5}}{4\times 13.03}\ \text{mV/m}$$

$$= \frac{\sqrt{300\times 500}}{4\times 13.03} \cong \frac{\cancel{400}^{100}}{4\times 13} = 7.4\ \text{mV/m}$$

$$\begin{array}{r} 7.6 \\ 13\,)\overline{100} \\ \underline{91} \\ 90 \\ 12 \end{array}$$

$F = 0.25$

i.e. the received voltage will be 0.25 of what it would have been with free-space propagation conditions.